

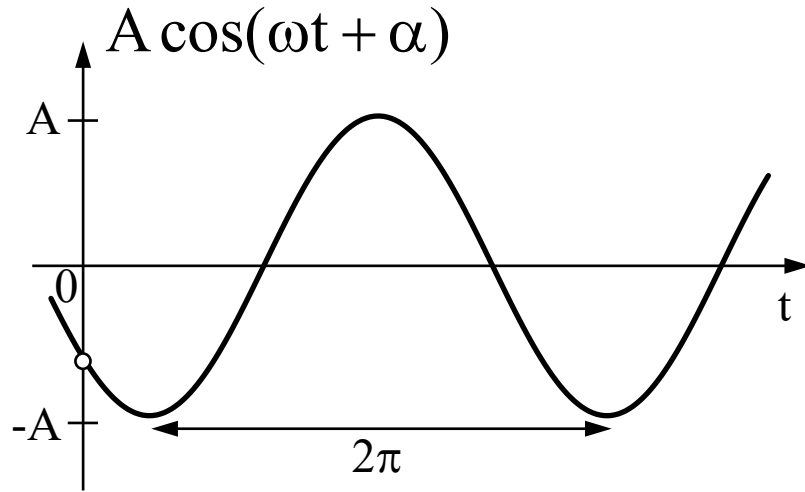
Machines électriques

Rappel sur les phaseurs

André Hodder

Grandeur sinusoïdale

Domaine temporel



Valeur instantanée

$$u \equiv u(t) = \hat{U} \cos(\omega t + \alpha) = \sqrt{2} U \cos(\omega t + \alpha)$$

$$i \equiv i(t) = \hat{I} \cos(\omega t + \beta) = \sqrt{2} I \cos(\omega t + \beta)$$

↑
valeur de crête

↑
valeur efficace

Phaseurs

$\xrightarrow{\text{Euler}}$

$$U(\cos(\omega t + \alpha) + j \sin(\omega t + \alpha)) = U e^{j\omega t} e^{j\alpha}$$

$$I(\cos(\omega t + \beta) + j \sin(\omega t + \beta)) = I e^{j\omega t} e^{j\beta}$$

Phaseurs

$$\underline{U} = U(\cos(\alpha) + j \sin(\alpha)) = U e^{j\alpha}$$

$$\underline{I} = I(\cos(\beta) + j \sin(\beta)) = I e^{j\beta}$$

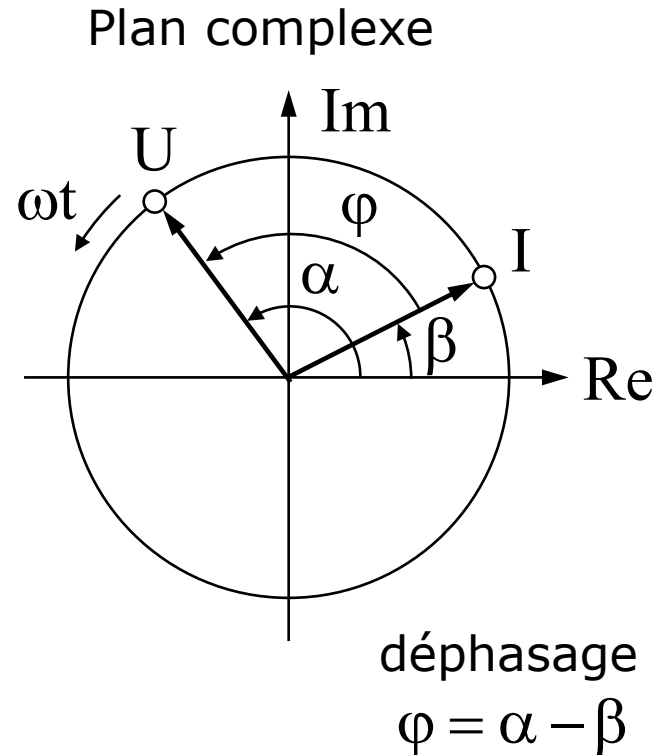
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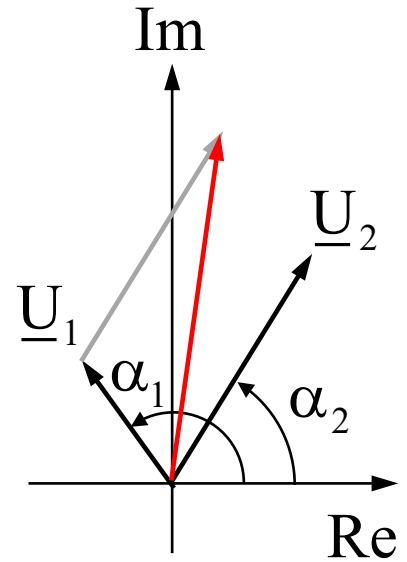
↑
valeur efficace



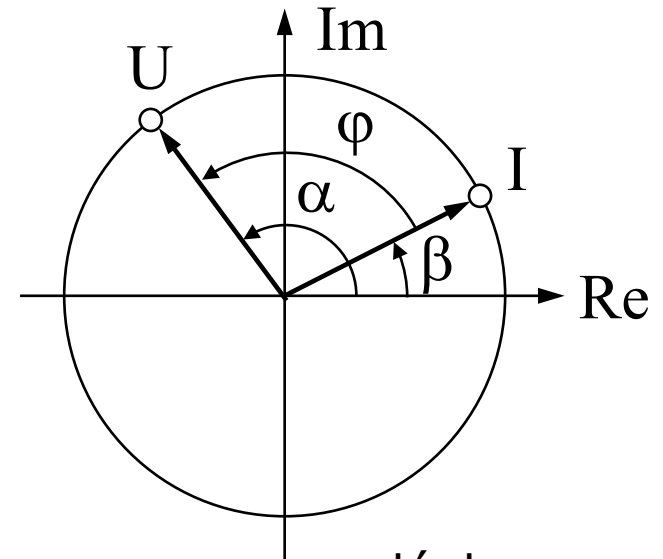
Phaseurs

Somme de phaseurs

$$\begin{aligned}\underline{U}_1 + \underline{U}_2 &= U_1 e^{j\alpha_1} + U_2 e^{j\alpha_2} \\ &= U_1 (\cos \alpha_1 + j \sin \alpha_1) + U_2 (\cos \alpha_2 + j \sin \alpha_2) \\ &= [U_1 \cos \alpha_1 + U_2 \cos \alpha_2] + j[U_1 \sin \alpha_1 + U_2 \sin \alpha_2]\end{aligned}$$



Phaseurs



Division de phaseurs

$$\frac{\underline{\underline{U}}}{\underline{\underline{I}}} = \frac{U e^{j\alpha}}{I e^{j\beta}} = \frac{U}{I} e^{j\alpha} e^{-j\beta} = \frac{U}{I} e^{j(\alpha-\beta)} = \frac{U}{I} e^{j(\varphi)} = Z e^{j(\varphi)} \quad \begin{array}{l} \text{déphasage} \\ \varphi = \alpha - \beta \end{array}$$

Impédance

$$|\underline{\underline{Z}}| = \frac{|\underline{\underline{U}}|}{|\underline{\underline{I}}|} = \frac{U}{I} = Z$$

Norme

$$\begin{aligned} |\underline{\underline{U}}| &= |U(\cos \alpha + j \sin \alpha)| \\ &= \sqrt{(\operatorname{Re}\{\underline{\underline{U}}\})^2 + (\operatorname{Im}\{\underline{\underline{U}}\})^2} \\ &= \sqrt{(U \cos \alpha)^2 + (U \sin \alpha)^2} \end{aligned}$$